Firm Cyclicality and Financial Frictions

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The views presented in this paper are our own and do not represent the views of Danmarks Nationalbank

Motivation

Financial frictions important in macro models

Gertler and Gilchrist (1994); Khan and Thomas (2013); Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999); Jermann and Quadrini (2012)

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Heterogeneous firm model:

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   - levels, growth rates, **cyclicality**
   - both real (employment, sales, ...) and financial (debt, assets, ...) variables
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   - document aze x size distribution of leverage
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   - *only young AND small grow their leverage*
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(Partial) literature review

- **Empirics: Cyclicality of firms by size and/or age**
  - Gertler and Gilchrist (1994); Moscarini and Postel-Vinay (2012); Fort et al. (2013); Crouzet and Mehrotra (2020); Gavazza et al. (2018) ...
  - Finance: Covas and Haan (2011); Chodorow-Reich (2014); Cloyne et al. (2019); Begenau and Salomao (2018); Crouzet and Mehrotra (2020), ...
  - Our contributions:
    (i) Registry data $\implies$ very young and all sizes, firm-level finance vars
    (ii) Cyclicalities by joint age-size for both real and financial variables
- **Empirics: Firm age/size dynamics over the lifecycle**
  - Haltiwanger et al. (2013); Dinlersoz et al. (2018); Sterk et al. (2021), ...
  - Our contribution: Financial averages over lifecycle by joint age-size
- **Models: Heterogeneous firm models with financial frictions**
  - Cooley and Quadrini (2001); Khan and Thomas (2013); Ottonello and Winberry (2020), ...
  - Our contribution: Het-firm business cycle model calibrated to firm age and size distributions, finance frictions vs returs to scale heterogeneity

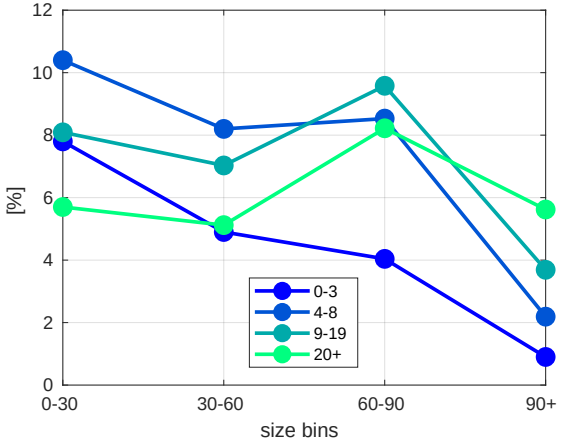
Data

- Administrative micro-level datasets, 2001-2019
- Tax information from SKAT + survey by Statistics Denmark
- 90,000 firms per year, cca 2M firm-year observations
- Variables:
  - employment (both headcount and FTE), sales, value added,...
  - debt, assets, equity
- Restrictions:
  - only private firms
  - non-finance sectors
  - max employment > 2
- All firm sizes
- All ages

details details

Age x Size

Firm shares



Estimating cyclicality

$$g^X_{i,t} = \sum_j \sum_k \left(\alpha_{j,k} + \beta_{j,k} g^{GDP}_t\right) \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \sum_l \left(\gamma_l + \delta_l g^{GDP}_t\right) \mathbb{1}_{i \in S_l} + \varepsilon_{i,t}$$

- g^X growth rate of variable of interest
- $\mathbb{1}_{i \in I_j}, \mathbb{1}_{i \in A_k}, \mathbb{1}_{I \in S_l}$: size, age and sectoral dummies
- $\beta_{j,k}$ "*cyclicality*" of X among firms of size j *and* age k
- controlling for sectoral differences in level and cyclicality of g^X
- new: interaction of size and age

Cyclicality of employment

$$\sum_j \sum_k \left(\alpha_{j,k} + \beta_{j,k} g_t^{GDP} \right) \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k}$$

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$$\sum_j \sum_k \left(\alpha_{j,k} + \beta_{j,k} g_t^{GDP}\right) \mathbb{1}_{i \in l_j} \mathbb{1}_{i \in A_k} \rightarrow \sum_j \left(\alpha_j + \beta_j \; g_t^{GDP}\right) \mathbb{1}_{i \in l_j}$$

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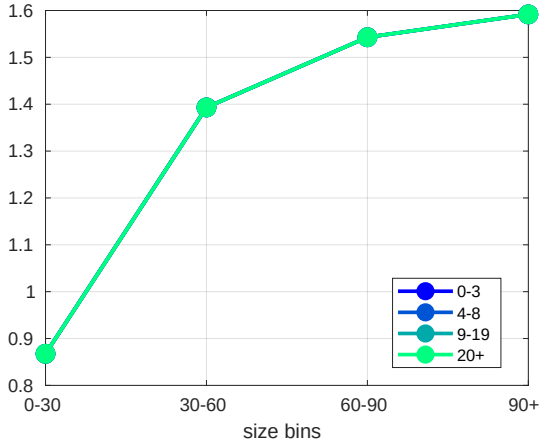

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Dinlersoz et al. (2018)

- No interaction $\rightarrow$ marginal effect of age the same in every size group

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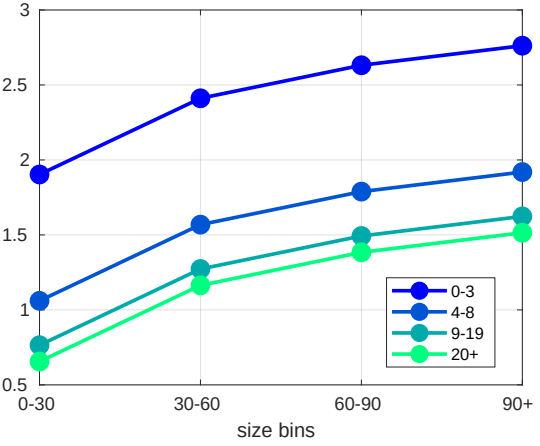

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- Younger and Larger firms more cyclica

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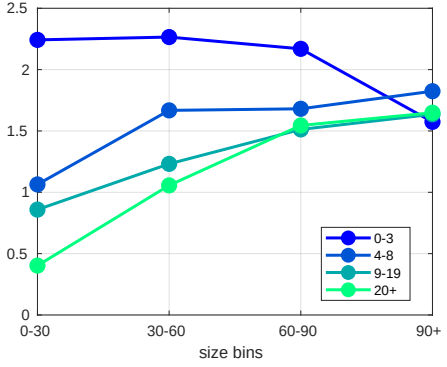

▸ sales ▸ assets, debt

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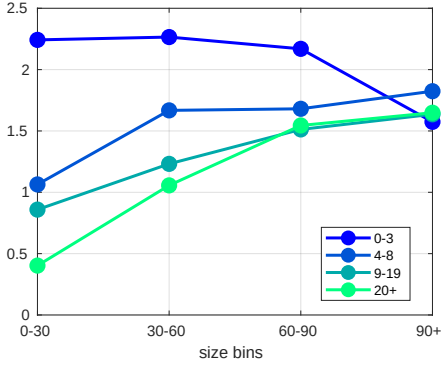

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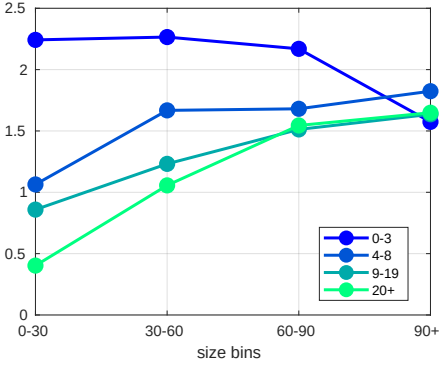

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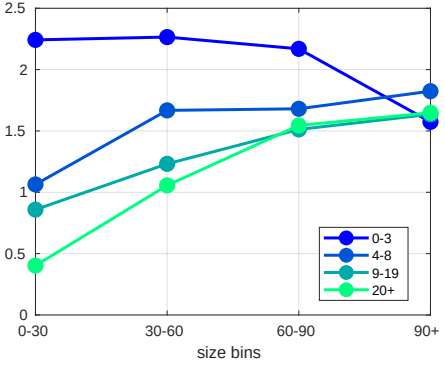

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$\Rightarrow$ large firms are much more alike than small firms

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Empirical results interpretation

1. Cyclicality of sales, employment, debt and assets
2. Document the size x age heterogeneity in the distribution of levels and growth rates of variables of interest
3. Effects of leverage on cyclicality
4. Returns to scale estimation

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Our interpretation: different forces operate along size and age

- cyclical worsening of financing hits young firms particularly hard
- large firms more exposed to aggregate business cycle

Model overview

Khan and Thomas (2013) to matching Age x Size firm distribution

- Firms produce using labour and rented capital
- Borrowing subject to collateral constraint
  firms postpone dividends to accumulate net worth
- Decreasing returns to scale ($\Rightarrow$finite optimal firm size)
- Calibration: firm types
  - finite number of productivity types
  - productivity penalty for entrants
  - superstar firms to match the very right tail
  - + **heterogeneity in returns to scale and starting networth**

Model experiments

1. Simple steady-state focused model does not match the data
   "Steady-state calibration"
   - setting:
     - no heterogeneity in rts
   - finding: no combination of MIT shocks to colateral constraint, interest rate and tfp can replicate the cyclicality results from the data

▸ calibration ▸ results

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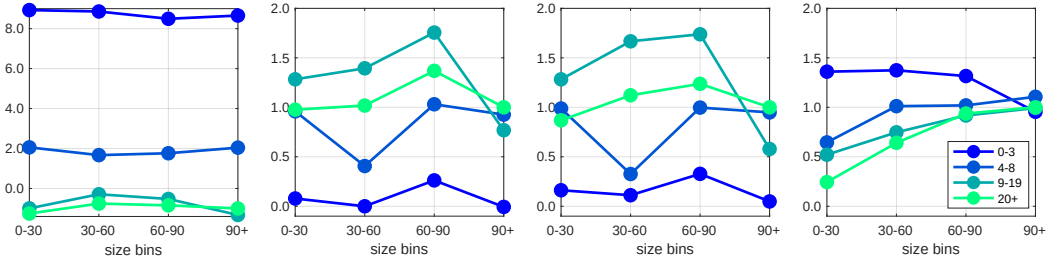
3. Policy experiments:
   - labor subsidy
   - debt relief

Model results

Cyclical response in the “steady state” calibration

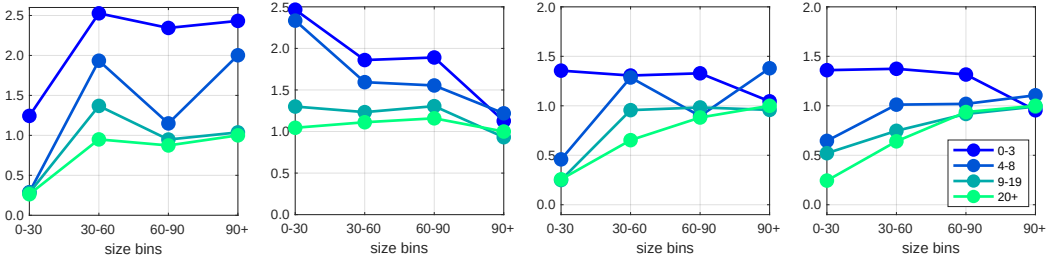


(a) Collateral ($\bar{\phi}$) (b) Discounts (r) (c) TFP (z) (d) Data

- Exercise: consider all possible shock that can hit firms and estimate the cyclicality along irf
- No individual shock (or combination) can generate positive gradient in cyclicality wrt size

Model results

Cyclical response in the “cyclical” calibration

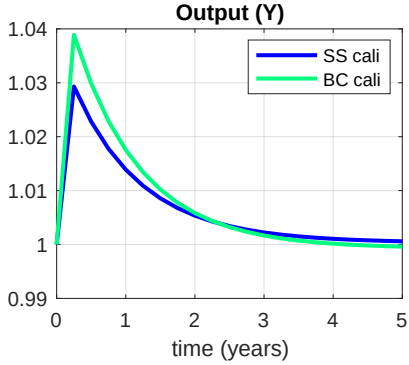


(a) Het. η_s (b) Het. n_s^e (c) Full model (d) Data

- Heterogeneous rts generates positive gradient wrt size!
  intuition: $y = zk^\alpha \Rightarrow k^* = (\alpha z/r)^{\frac{1}{1-\alpha}} \Rightarrow \frac{\partial \log k}{\partial \log z} = \frac{1}{1-\alpha} > 0$
- Heterogenous starting net worth needed to shorten convergence to optimal size

Policy experiments

Labor subsidy



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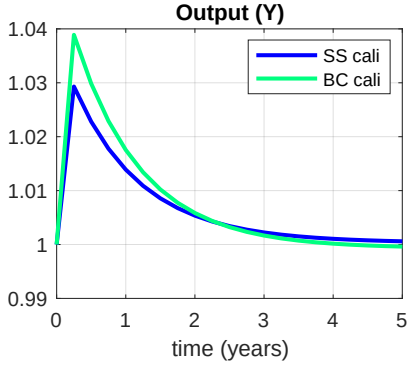

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  - cyclical calibration: large firms react more, regardless of age

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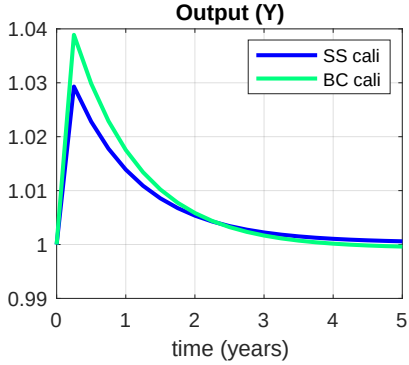

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- Large firms responding more $\Rightarrow$ larger aggregate response

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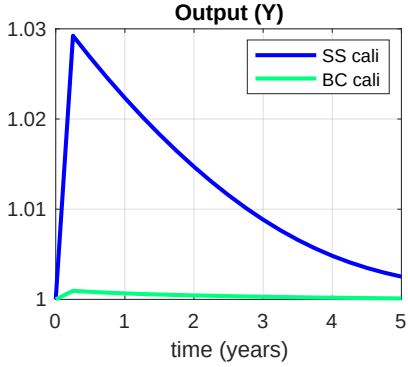
Debt relief



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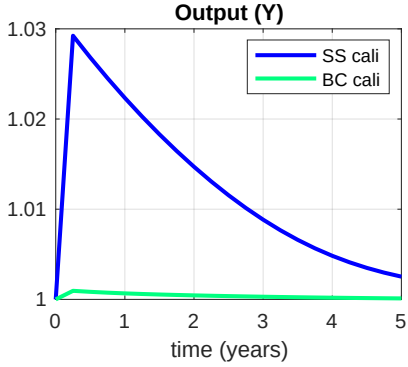

- SS calibration much more powerfull
- Why? Debt relief helps only constrained firms young, large not constrained in cyclical calibration

Conclusion

1. Empirics: Size is not a perfect proxy for age and vice versa
   - Size gradient of cyclicality depends on age (youngest↓, others↑)
   - in terms of cyclicality, large firms are more alike than small firms
2. Model
   - financial frictions make young firms highly responsive to shocks
   - but firms grow out of financial constraint relatively fast → second mechanism is needed to get the cyclicality by size right
     - **entrants**: positive corelation of starting net-worth with productivity generates negative cyclicality gradient wrt size
     - **older firms**: heterogeneous returns to scale generate positive cyclicality gradient with respect to size
3. Policy implications
   - who responds drives the aggregate reaction
     → capturing the behaviour of young-large firms particularly important

Thank you!

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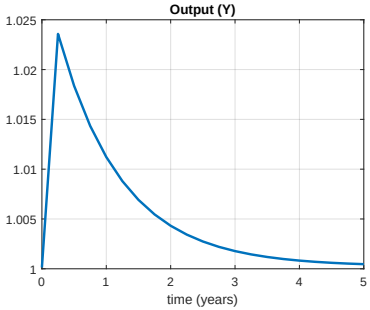
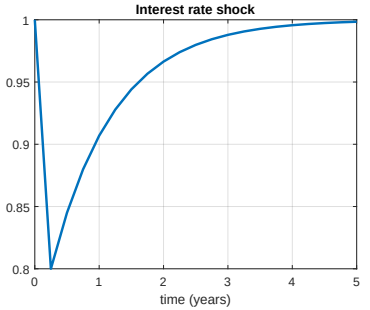
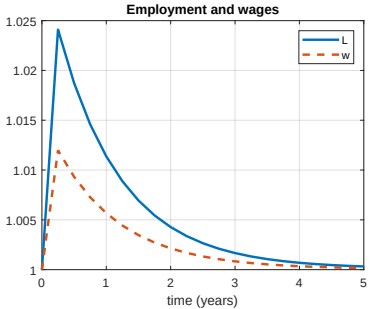
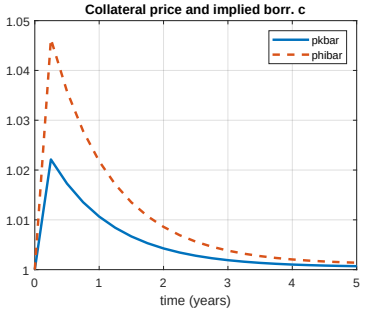
Monetary policy experiment

Aggregates



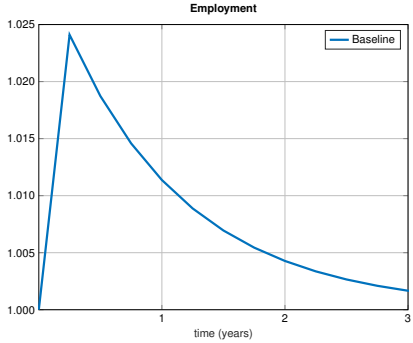


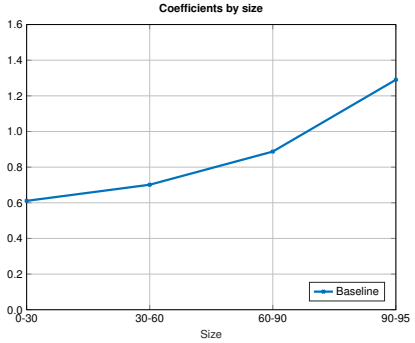


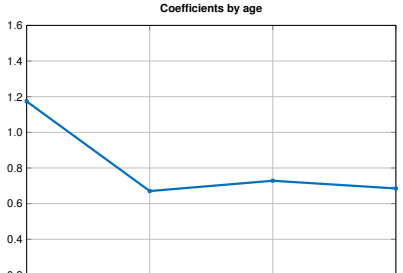


Monetary policy experiment

Counterfactuals



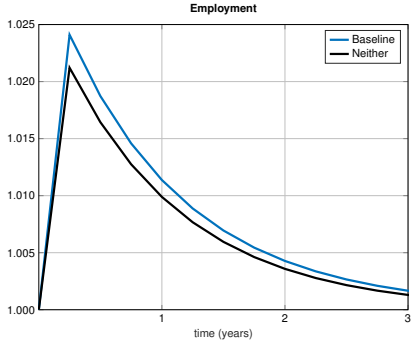


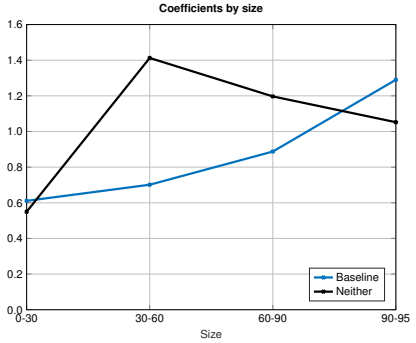


- Baseline = HRTS+finance

Monetary policy experiment

Counterfactuals

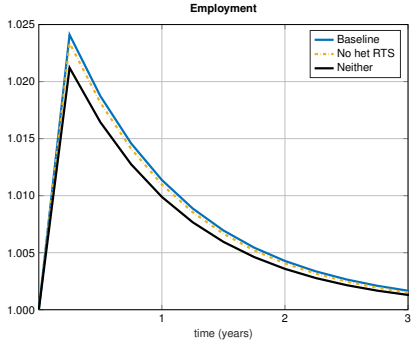


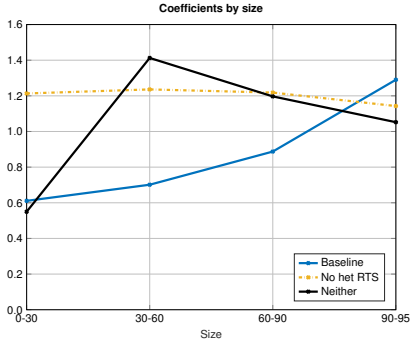


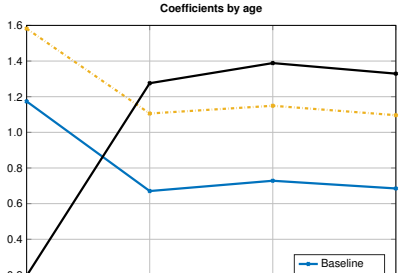
- Baseline = HRTS+finance
- neither: less amplification, age cyclicality wrong

Monetary policy experiment

Counterfactuals



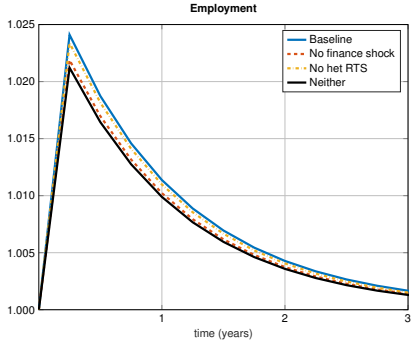


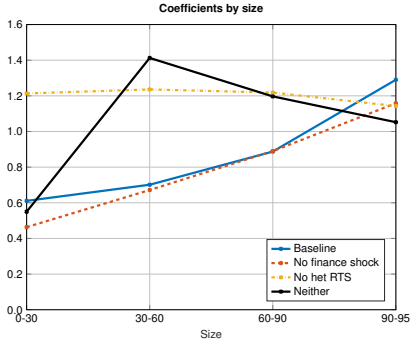


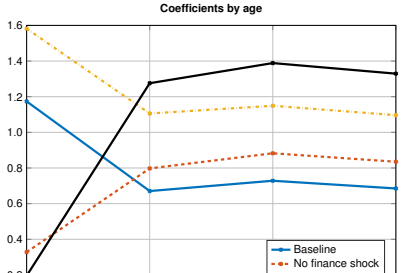
- Baseline = HRTS+finance
- neither: less amplification, age cyclicality wrong
- only finance: flat size

Monetary policy experiment

Counterfactuals



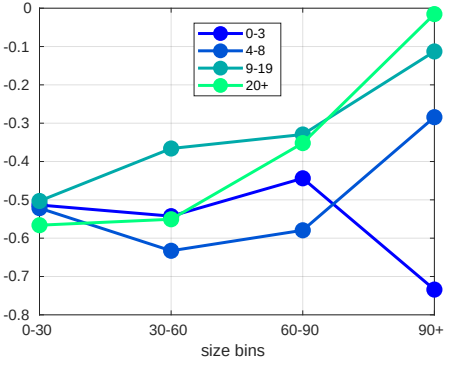




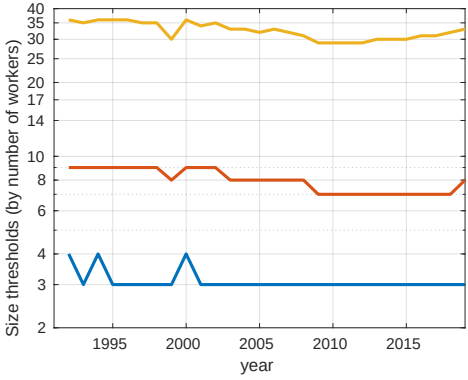
- Baseline = HRTS+finance
- neither: less amplification, age cyclicality wrong
- only finance: flat size

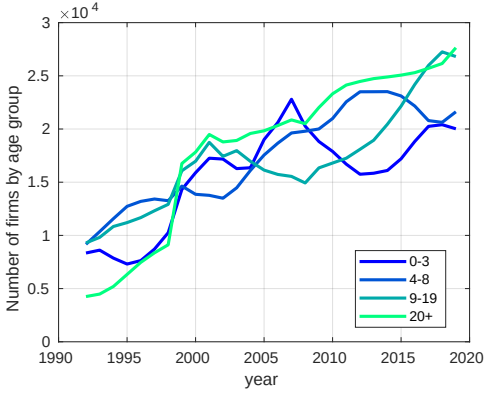
Predicted cyclicality

DA



Size and Age





- Small firms really small
- relative size of age groups cyclical
- base line results starting from 2001
  - number of firms lower in 90's → possible selection issues ?
  - results robustish when using the 90's

Data

Averages of Variables of Interest by Age and Size

	Age groups				Size groups			
	0-3	4-8	9-19	20+	0-30	30-60	60-90	90+
Employment	9.8	13.6	21.0	40.6	1.7	4.4	12.4	130.0
Sales	20993	32895	54765	121159	4857	10885	28497	363759
Assets	18172	32652	57960	141251	11022	19999	25445	375965
Debt	11590	19247	32340	74278	5553	11175	13917	208246
Equity	6581	13405	25620	66972	5469	8824	11527	167715
Bank loans	1128	2539	3970	8317	672	1059	2251	23699
Equity<0	47.8	46.8	46.7	46.8	47.0	47.0	46.9	46.7
Bank loans>0	50.3	60.1	63.9	68.2	48.0	59.8	69.1	80.4
D/A	0.85	0.79	0.70	0.62	0.76	0.75	0.73	0.68
C/A	0.17	0.16	0.15	0.11	0.18	0.15	0.13	0.09

Note: Sales, assets, debt and equity in thousands of DKK (1000 DKK = 134 EUR $\approx$ 150-200 USD). Debt/assets (DA). Continuing firms only.

back

Data

Number of observations by size and age

	Age							
	0	[1,5)	[5,10)	[10,15)	[15,20)	[20,25)	25+	all
emp<50	0.98	0.98	0.96	0.94	0.91	0.9	0.84	0.94
emp>=50	0.02	0.02	0.04	0.06	0.09	0.10	0.16	0.06
size(0-30)	0.46	0.41	0.35	0.30	0.26	0.26	0.23	0.33
size(30-60)	0.25	0.28	0.26	0.24	0.23	0.23	0.19	0.25
size(60-90)	0.23	0.26	0.31	0.33	0.35	0.33	0.32	0.30
size(90+)	0.06	0.06	0.09	0.13	0.16	0.18	0.26	0.12
all	17273	236500	168725	117111	78781	55854	115105	789349

Note: size defined on headcount employment to prevent to much switches due to hours fluctuations

back

Regression framework

Goal: recover AGE $\times$ SIZE interaction

Regression framework

Goal: recover AGE $\times$ SIZE interaction

- A: age groups (0-3, 4-7, 8-19, 20+)
- I: employment groups (percentile cutoffs: 30,60,90,95)

details

Regression framework

Goal: recover AGE $\times$ SIZE interaction for levels, growth rates,

$$X_{i,t} = \sum_j \sum_k \alpha_{j,k} \, \mathbb{1}_{i \in l_j} \mathbb{1}_{i \in A_k} + \varepsilon_{i,t}$$

- A: age groups (0-3, 4-7, 8-19, 20+)
- l: employment groups (percentile cutoffs: 30,60,90,95)
- $X_{i,t}$: $i-th$ firm variable of interest

details

Regression framework

Goal: recover AGE $\times$ SIZE interaction
for levels, growth rates, cyclicality,

$$X_{i,t} = \sum_j \sum_k \alpha_{j,k} \, \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \varepsilon_{i,t}$$

$$g^X_{i,t} = \sum_j \sum_k \left(\alpha_{j,k} + \beta_{j,k} g^Y_t \right) \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \varepsilon_{i,t}$$

- A: age groups (0-3, 4-7, 8-19, 20+)
- I: employment groups (percentile cutoffs: 30,60,90,95)
- $X_{i,t}$: $i-th$ firm variable of interest
- $g^X_{i,t}$: (normalised) growth rate of $X_{i,t}$
- g^Y_t: GDP growth rate

details

Regression framework

Goal: recover AGE $\times$ SIZE interaction
for levels, growth rates, cyclicality, effect of leverage

$$X_{i,t} = \sum_j \sum_k \alpha_{j,k} \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \varepsilon_{i,t}$$

$$g^X_{i,t} = \sum_j \sum_k \left(\alpha_{j,k} + \beta_{j,k} g^Y_t \right) \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \varepsilon_{i,t}$$

$$g^X_{i,t} = \sum_j \sum_k \left(\alpha_{j,k} + \beta_{j,k} g^Y_t \right) \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \varepsilon_{i,t}$$
$$+ \sum_m (\omega_m + \psi_m y_t) \mathbb{1}_{i \in DA_m}$$

- A: age groups (0-3, 4-7, 8-19, 20+)
- I: employment groups (percentile cutoffs: 30,60,90,95)
- $X_{i,t}$: $i-th$ firm variable of interest
- $g^X_{i,t}$: (normalised) growth rate of $X_{i,t}$
- g^Y_t: GDP growth rate
- DA_{it}: quintile of leverage distribution

details

Regression framework

Goal: recover AGE $\times$ SIZE interaction
for levels, growth rates, cyclicality, effect of leverage

$$X_{i,t} = \sum_j \sum_k \alpha_{j,k} \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \sum_l \gamma_l \mathbb{1}_{i \in S_l} + \varepsilon_{i,t}$$

$$g^X_{i,t} = \sum_j \sum_k \left(\alpha_{j,k} + \beta_{j,k} g^Y_t\right) \mathbb{1}_{i \in I_j} \mathbb{1}_{i \in A_k} + \sum_l \left(\gamma_l + \delta_l g^Y_t\right) \mathbb{1}_{i \in S_l} + \varepsilon_{i,t}$$

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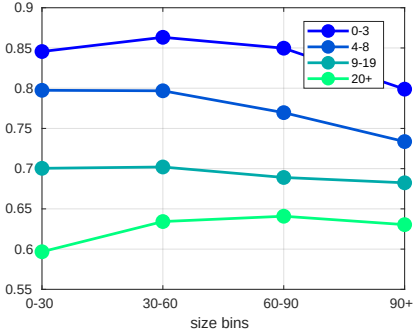
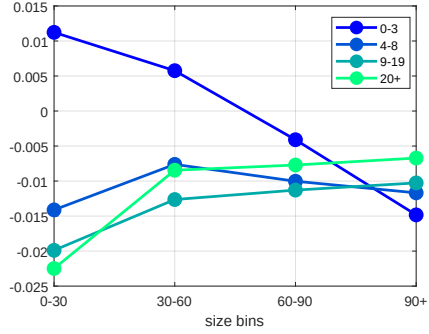
- A: age groups (0-3, 4-7, 8-19, 20+)
- I: employment groups (percentile cutoffs: 30,60,90,95)
- $X_{i,t}$: $i-th$ firm variable of interest
- $g^X_{i,t}$: (normalised) growth rate of $X_{i,t}$
- g^Y_t: GDP growth rate
- DA_{it}: quintile of leverage distribution
- S: 36 sectors

details

Level and growth rate of Leverage

$$\hat{g}_{x_{i,t}} \text{ or } x_{i,t} = \sum_m \omega_m \mathbb{1}_{i \in DA(m)} + \sum_l \gamma_l \mathbb{1}_{i \in S(l)} \quad (1)$$





- Debt/Asset ratio is generally falling with age,
- Both debt and assets growing $\Rightarrow$ assets are growing faster than debt
- However, for small AND young DA is increasing
  our interpretation: small AND young cannot borrow as much as they want

Empirical results overview

1. Cyclicality of employment, sales, debt and assets
   - conditional on size: young $>$ old
   - conditional on age: large $>$ small for most, but opposite for entrants
2. Document the size x age heterogeneity in the distribution of levels and growth rates of variables of interest
   - unlike capital in models, assets differ by age in each size category
   - *on average*, only very young firms grow (strongly decreasing in size), firms above 10 shrink (weakly increasing in size)
   - Over age, both debt and assets increase
     - for most firms, assets grow faster than debt $\Rightarrow$ D/A $\downarrow$
     - for young and small, debt grows faster $\Rightarrow$ D/A $\uparrow$

▸ results

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▸ results

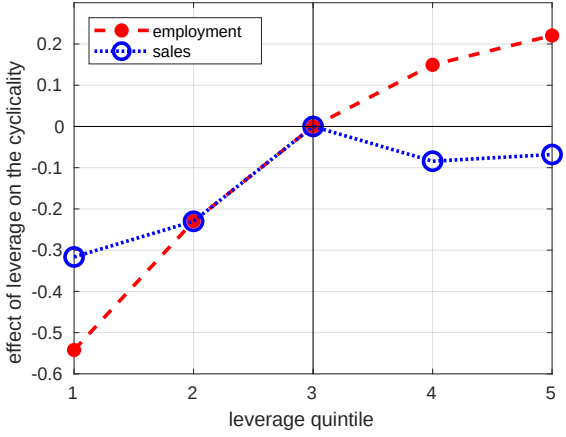
3. Effects of leverage on cyclicality
   - differences bettwen employment and sales suggesting finance can mitigate demand shocks
   - effect of leverage at least partly independent of Size x Age controls

Effect of Leverage I

$$\hat{g}_{x_{i,t}} = \sum_{m} (\omega_m + \psi_m g_t^{GDP}) \mathbb{1}_{i \in DA(m)} + \sum_{l} (\gamma_l + \delta_l g_t^{GDP}) \mathbb{1}_{i \in S(l)} \tag{2}$$

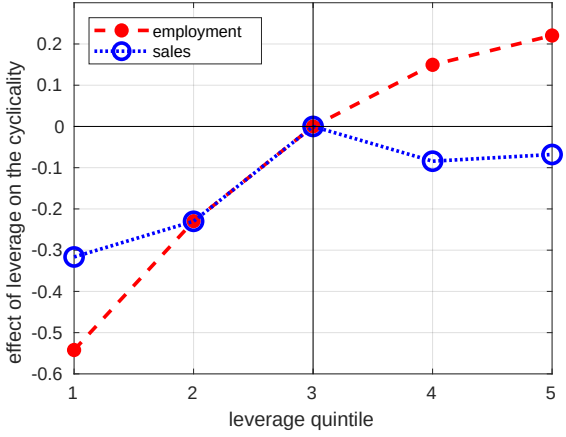
Effect of Leverage I

$$\hat{g}_{x_{i,t}} = \sum_m (\omega_m + \psi_m g_t^{GDP}) \mathbb{1}_{i \in DA(m)} + \sum_l (\gamma_l + \delta_l g_t^{GDP}) \mathbb{1}_{i \in S(l)} \quad (2)$$



Effect of Leverage I

$$\hat{g}_{x_{i,t}} = \sum_m (\omega_m + \psi_m g_t^{GDP}) \mathbb{1}_{i \in DA(m)} + \sum_l (\gamma_l + \delta_l g_t^{GDP}) \mathbb{1}_{i \in S(l)} \quad (2)$$

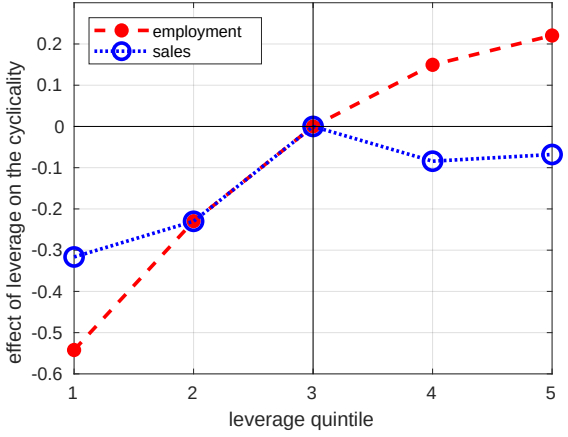


Effect of Leverage on cyclicality:

- Employment: ↑
- Sales: ?

Effect of Leverage I

$$\hat{g}_{x_{i,t}} = \sum_m (\omega_m + \psi_m g_t^{GDP}) \mathbb{1}_{i \in DA(m)} + \sum_l (\gamma_l + \delta_l g_t^{GDP}) \mathbb{1}_{i \in S(l)} \quad (2)$$



Effect of Leverage on cyclicality:

- Employment: ↑
- Sales: ?

Interpretation: finance used to mitigate umployment response of consumer demand shocks

Effect of Leverage II

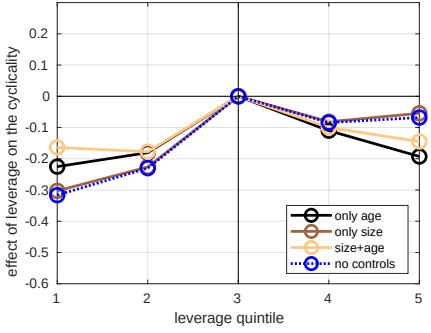
Effect of Size x Age controls

$$\hat{g}x_{i,t} = \sum_{m}(\omega_m + \psi_m y_t)\mathbb{1}_{i\in DA(m)} + \sum_{j}\sum_{k}(\alpha_{j,k} + \beta_{j,k}y_t)\mathbb{1}_{i\in I_t^j}\mathbb{1}_{i\in A(k)} + \sum_{l}(\gamma_l + \delta_l y_t)\mathbb{1}_{i\in S(l)}$$

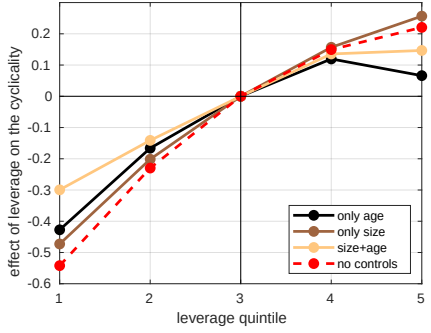
Effect of Leverage II

Effect of Size x Age controls

$$\hat{g}x_{i,t} = \sum_m (\omega_m + \psi_m y_t)\mathbb{1}_{i \in DA(m)} + \sum_j \sum_k (\alpha_{j,k} + \beta_{j,k} y_t)\mathbb{1}_{i \in I_t^j}\mathbb{1}_{i \in A(k)} + \sum_l (\gamma_l + \delta_l y_t)\mathbb{1}_{i \in S(l)}$$



(a) Sales: + Controls

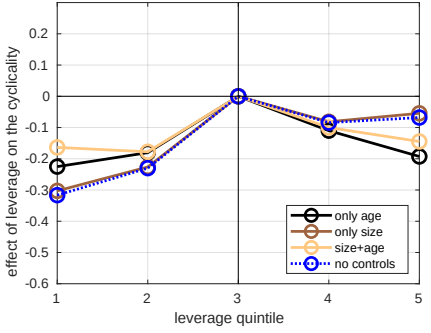


(b) Employment: + Controls

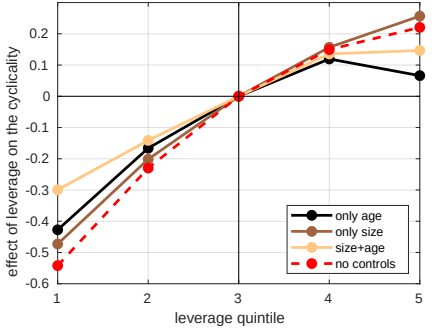
Effect of Leverage II

Effect of Size x Age controls

$$\hat{g}x_{i,t} = \sum_m (\omega_m + \psi_m y_t)\mathbb{1}_{i \in DA(m)} + \sum_j \sum_k (\alpha_{j,k} + \beta_{j,k} y_t)\mathbb{1}_{i \in I_t^j}\mathbb{1}_{i \in A(k)} + \sum_l (\gamma_l + \delta_l y_t)\mathbb{1}_{i \in S(l)}$$



(a) Sales: + Controls

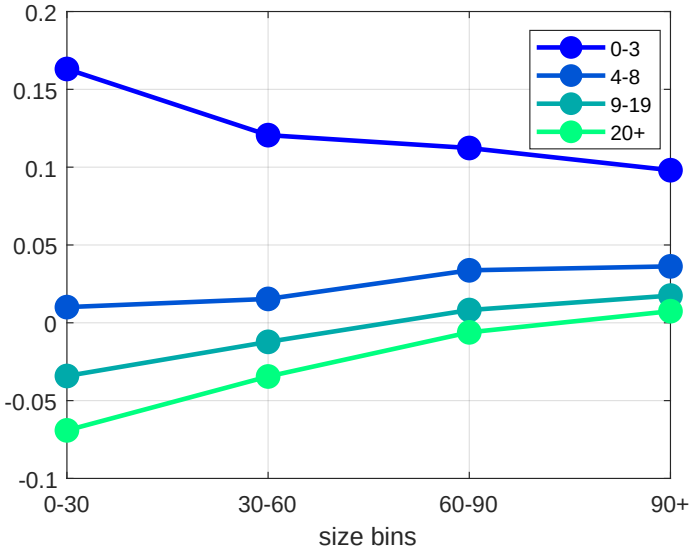


(b) Employment: + Controls

- Adding controls does no change the results qualitatively

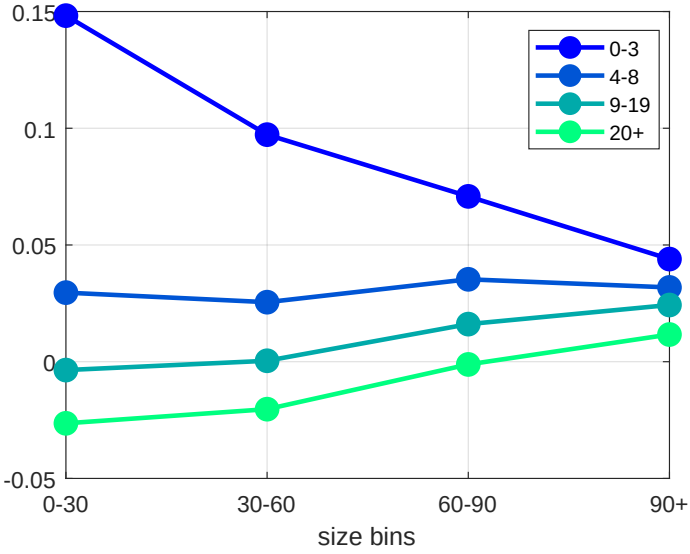
Average growth rate of sales

Basic moments

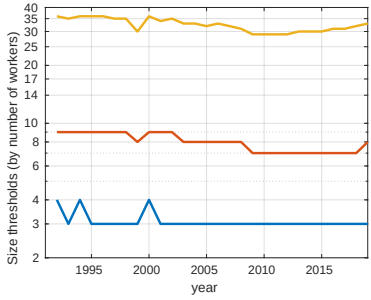


Average growth rate of assets

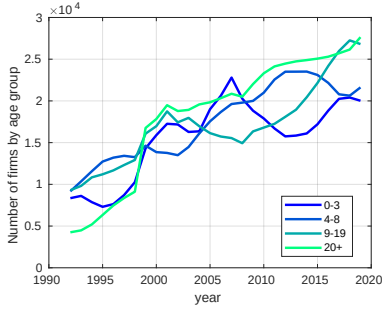
Basic moments



Size Thresholds and Number of Firms in Different Age Bins



(a) Size thresholds (log scale on y-axis)

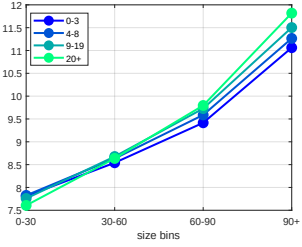


(b) Number of firms in each age group over time

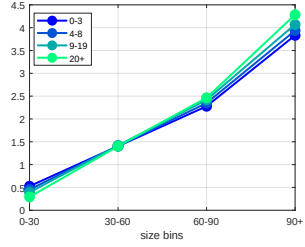
back

Basic moments

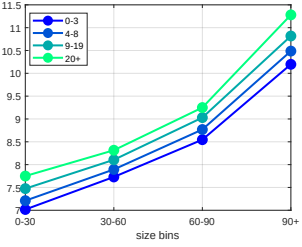
Figure: Average levels and growth rates by size and age



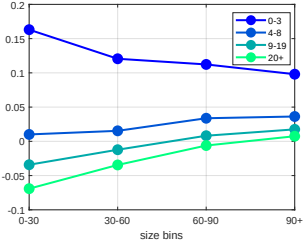
(a) (log) Sales



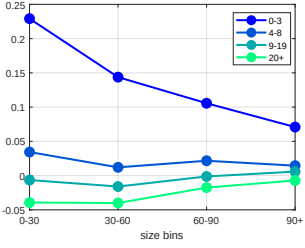
(b) (log) Employment



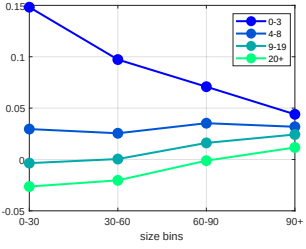
(c) (log) Assets



(d) Growth rate of sales



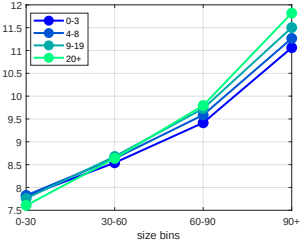
(e) Growth rate of employment



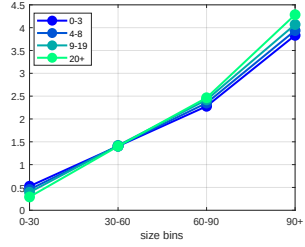
(f) Growth rate of assets

Basic moments

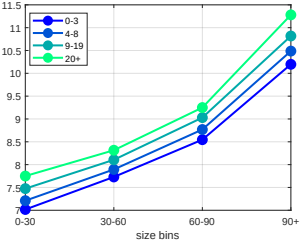
Figure: Average levels and growth rates by size and age



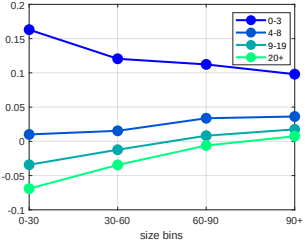
(a) (log) Sales



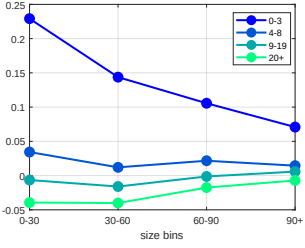
(b) (log) Employment



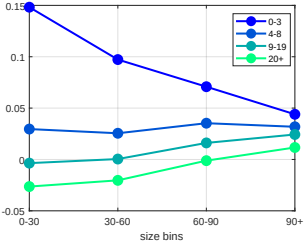
(c) (log) Assets



(d) Growth rate of sales



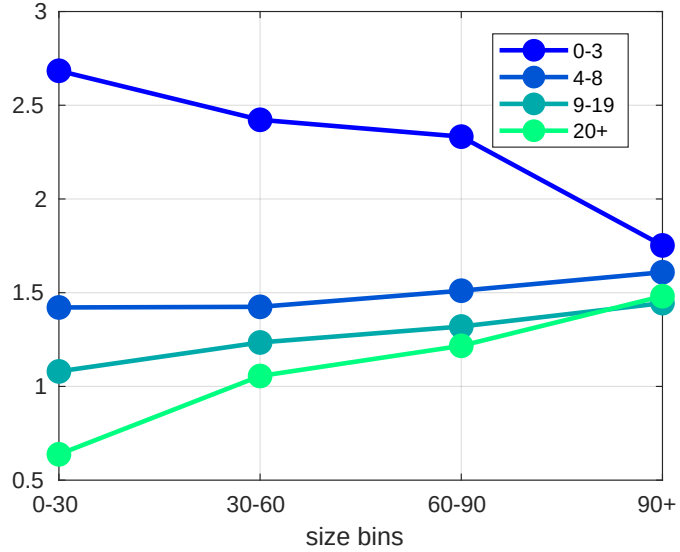
(e) Growth rate of employment



(f) Growth rate of assets

Predicted cyclicality

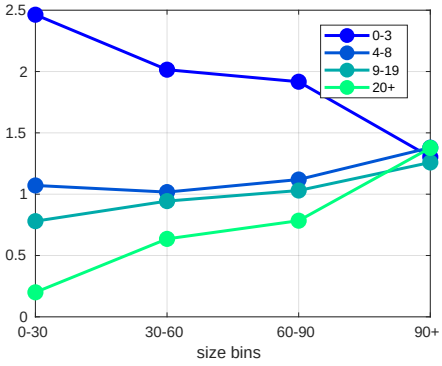
Sales

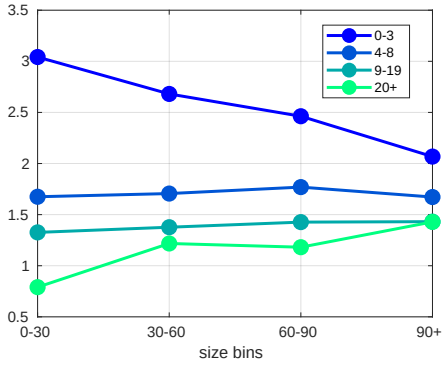


Same pattern:

Predicted cyclicality

Assets, Debt





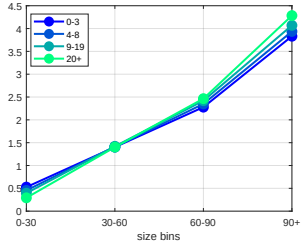
Same pattern:

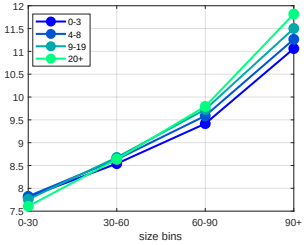
- entrants more cyclical
- entrants' cyclicality decreasing in size
- for everybody else cyclicality is (weakly) increasing with size

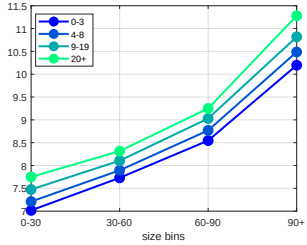
back

Basic moments

Levels of employment, sales and assets

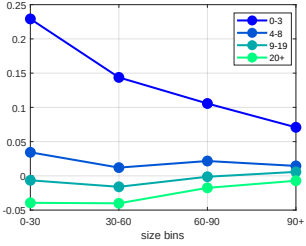


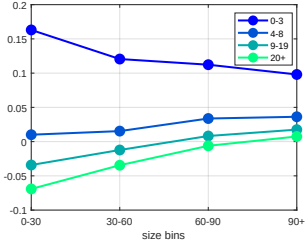


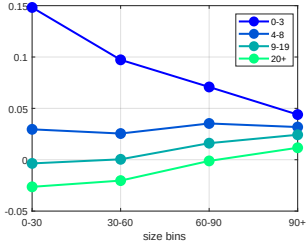


Basic moments

Growth rates of employment, sales and assets







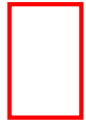
back

Effects of Leverage

- Growth rate

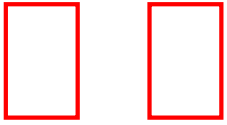
Effects of Leverage

- Growth rate
  - Emp: $\cap$



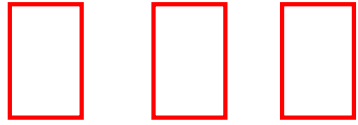
Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$



Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$
  - Assets: $-$

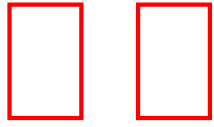


Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$
  - Assets: $-$
- Cyclicality

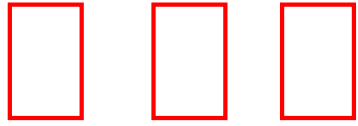
Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$
  - Assets: $-$
- Cyclicality
  - Emp+Assets: $+$



Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$
  - Assets: $-$
- Cyclicality
  - Emp+Assets: $+$
  - Sales: 0



Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$
  - Assets: $-$
- Cyclicality
  - Emp+Assets: $+$
  - Sales: 0
- Leverage vs Age x Size

Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$
  - Assets: $-$
- Cyclicality
  - Emp+Assets: $+$
  - Sales: 0
- Leverage vs Age x Size
  - growth rate coefs not (much) affected
    (apart from the least levered firms)

Effects of Leverage

- Growth rate
  - Emp: $\cap$
  - Sales: $+$
  - Assets: $-$
- Cyclicality
  - Emp+Assets: $+$
  - Sales: 0
- Leverage vs Age x Size
  - growth rate coefs not (much) affected
    (apart from the least levered firms)

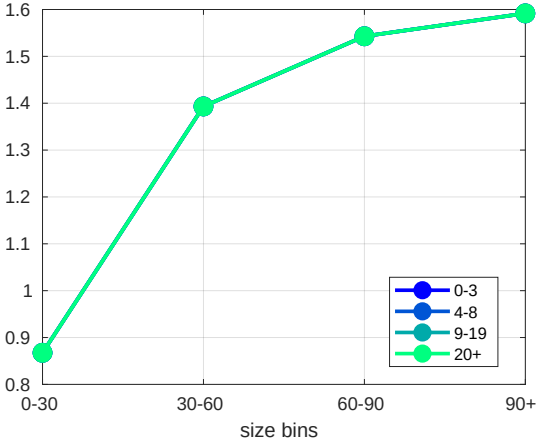
$\Rightarrow$ Leverage and Size x Age not (perfect) proxies

back

Cyclicality of employment

Only size

$$g_{i,t}^{emp} = \sum_j \left(\alpha_j + \beta_j g_t^Y\right) \mathbb{1}_{i \in I_j} + \sum_l \left(\gamma_l + \delta_l g_t^Y\right) \mathbb{1}_{i \in S_l} + \varepsilon_{i,t}$$

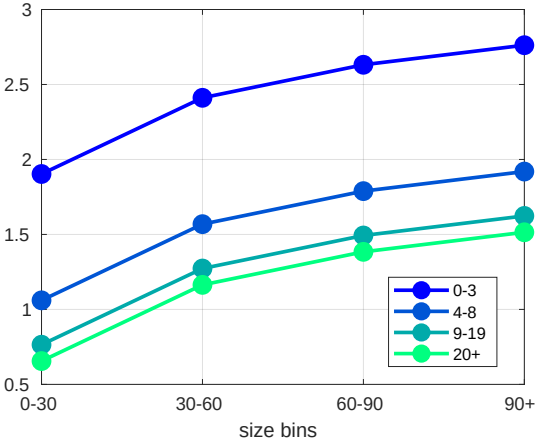


(similar framework to Crouzet and Mehrotra (2020))

Cyclicality of employment

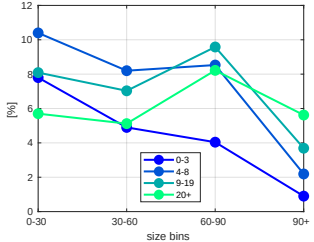
Additive size and age

$$g_{i,t}^{emp} = \sum_j \left(\alpha_j + \beta_j g_t^Y\right) \mathbb{1}_{i \in I_j} + \sum_k \left(\alpha_k + \beta_k g_t^Y\right) \mathbb{1}_{i \in A_k} + \sum_l \left(\gamma_l + \delta_l g_t^Y\right) \mathbb{1}_{i \in S_l} + \varepsilon_{i,t}$$

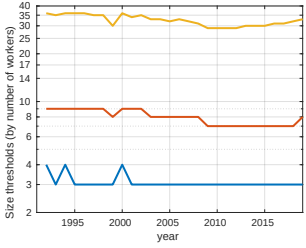


(similar framework to Dinlersoz et al. (2018))

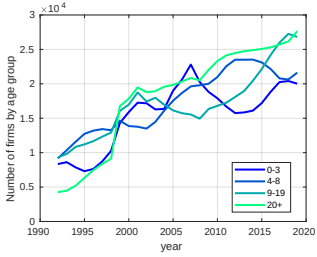
Firms size/age bins



(a) Bin sizes



(b) Size thresholds (log scale)



(c) Number of firms in each age group

Heterogeneity in returns to scale

RTS estimation:

1. three size groups: 0-7, 7-25, 25+, only non-transitioning firms
2. estimate $y = k^{\alpha} l^{\beta}$ using OP and LP with/without ACF
   either sectoral FE or sector-specific αs and βs
3. rts$= \alpha + \beta$

Heterogeneity in returns to scale

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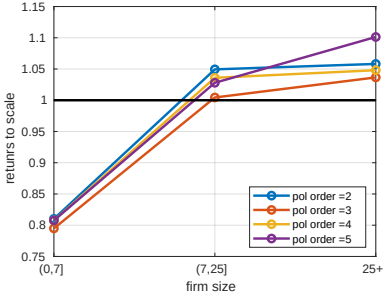
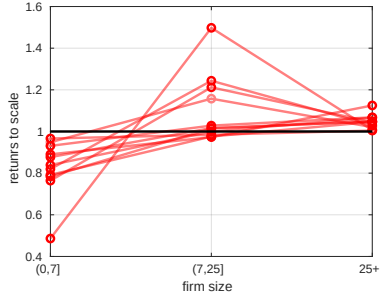


(a) Aggregate

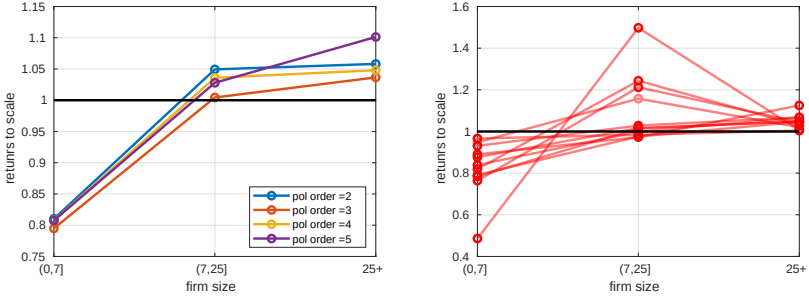


(b) Sector level

Heterogeneity in returns to scale

RTS estimation:

1. three size groups: 0-7, 7-25, 25+, only non-transitioning firms
2. estimate $y = k^{\alpha} l^{\beta}$ using OP and LP with/without ACF
   either sectoral FE or sector-specific αs and βs
3. rts$= \alpha + \beta$



(a) Aggregate (b) Sector level

- large firms returns to scale $>$ small firms
  $\oplus$: somewhat robust to estimation methodology,
  $\ominus$: medium size ?

Effect of Leverage III

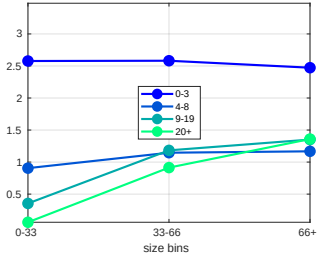
Effect of Size x Age controls II

$$\hat{g}_{x_{i,t}} = \sum_m \sum_j \sum_k (\alpha_{j,k,m} + \beta_{j,k,m} y_t) \mathbb{1}_{i \in I_t^j} \mathbb{1}_{i \in A(k)} \mathbb{1}_{i \in DA(m)} + \sum_l (\gamma_l + \delta_l y_t) \mathbb{1}_{i \in S(l)}$$

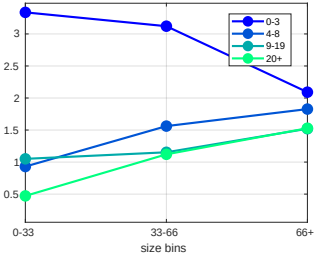
Effect of Leverage III

Effect of Size x Age controls II

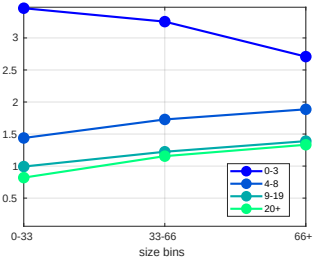
$$\hat{g}_{x_{i,t}} = \sum_m \sum_j \sum_k (\alpha_{j,k,m} + \beta_{j,k,m} y_t) \mathbb{1}_{i \in I_t^j} \mathbb{1}_{i \in A(k)} \mathbb{1}_{i \in DA(m)} + \sum_l (\gamma_l + \delta_l y_t) \mathbb{1}_{i \in S(l)}$$



(a) Lowest D/A



(b) Middle D/A

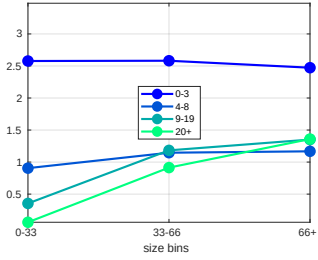


(c) Highest D/A

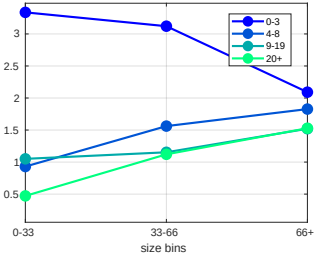
Effect of Leverage III

Effect of Size x Age controls II

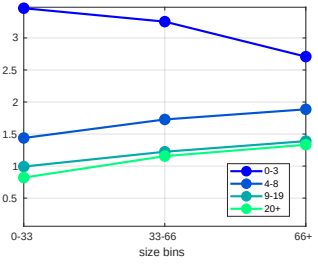
$$\hat{g}_{x_{i,t}} = \sum_m \sum_j \sum_k (\alpha_{j,k,m} + \beta_{j,k,m} y_t) \mathbb{1}_{i \in I_t^j} \mathbb{1}_{i \in A(k)} \mathbb{1}_{i \in DA(m)} + \sum_l (\gamma_l + \delta_l y_t) \mathbb{1}_{i \in S(l)}$$



(a) Lowest D/A



(b) Middle D/A



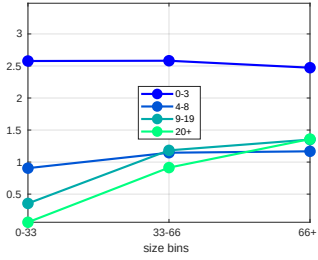
(c) Highest D/A

- Largest effects for smallest firms
- qualitative shape not affected

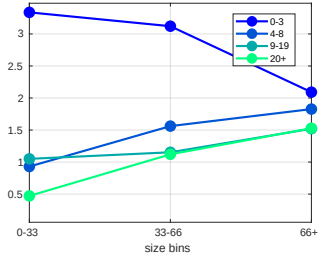
Effect of Leverage III

Effect of Size x Age controls II

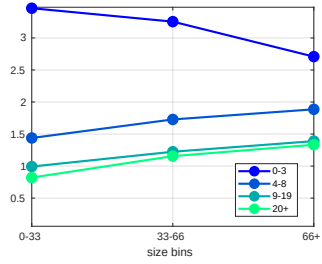
$$\hat{g}_{x_{i,t}} = \sum_m \sum_j \sum_k (\alpha_{j,k,m} + \beta_{j,k,m} y_t) \mathbb{1}_{i \in I_t^j} \mathbb{1}_{i \in A(k)} \mathbb{1}_{i \in DA(m)} + \sum_l (\gamma_l + \delta_l y_t) \mathbb{1}_{i \in S(l)}$$



(a) Lowest D/A



(b) Middle D/A



(c) Highest D/A

- Largest effects for smallest firms
- qualitative shape not affected
- our interpretation: Leverage capturing something beyond just Size x Age

$$\dot{n} = \left(\frac{\pi(k, s, g)}{k} - (\delta + r) p_K \right) k + rn - d \tag{3}$$

- d: dividend payout flow
- $d \geq 0$, firms cannot raise equity at all after the moment of birth
- firms payout dividends only when net worth exceeds an exogenous level $\bar{n}$, and payout such that net worth remains at $\bar{n}$

$\Rightarrow$ Firms therefore pay no dividends while they are young, but then start paying out dividends when they are older and have achieved sufficient scale.

- Production function: $q_i = z_i f(k_I, l_i)^{\eta_i}$
- Aggregate output $Q = \left(\int_0^G q_i^\theta \, \mathrm{d}i \right)^{\frac{1}{\theta}}$
  $\Rightarrow$ i-th firm demand: $q_i = p_i^{-1/(1-\theta)} Q$
- Profit function:

$$\pi = \max \left\{ q^\theta Q^{1-\theta} - wl \right\}$$

- Leontief production function $f(k, l) = \min\{k, \frac{l}{\alpha}\}$ implies

$$l^*(k) = \alpha k$$
$$\pi(k, s, g) = z^\theta k^{\eta\theta} Q^{1-\theta} - \alpha w k.$$

- Firm borrow at rate r s.t. $b \leq p_K \lambda k$

Model - Borrowing and collateral prices

- Borrowing constraint $b \leq \lambda \underline{p}_k k$
- Collateral price of capital $\underline{p}_k \leq p_k$
- Leverage: $\phi = \frac{p_k k}{p_k k - b}$
  - $p_k = 1 + \psi_k(1/K - \delta)$ $\quad (= 1$ in SS$)$
- Outcome:
  - endogenous leverage constraint $\phi \leq \bar{\phi} = \frac{p_k}{p_k - \lambda \underline{p}_k}$
  - fix for counterfactual behaviour of unconstrained firms
- Motivation:
  - banks less efficient in reselling capital in the case of default
  - reselling harder in recessions (Lanteri, 2018)

- Firm type $s \in \{1, 2, \ldots, S\}$
  - rts η_s
  - starting net worth
  - productivity z_S
- idiosyncratic productivity $z^J \in \{1, \ldots, J\}$ with transition matrix π^J
- entrant productivity penalty z_G
  - $g \in 1, 2$
  - all first start with $g = 1$, associated with productivity $z_1 < 1$ and exit rate penalty ξ_1
  - exogenous rate α_G firms transition to $g = 2$ associated with $z_2 = 1$ and $\xi_2 < \xi_1$
- additionaly z^* superstar firms
- $z = z_s^S z_g^G z_j^J$

$$
\begin{aligned}
rv(n,s,g,j) = \max_{0\le p_K k\le \bar{\phi} n} & d(n) + v_n(n,s,g,j)\left(\left(\frac{\pi(k,s,g,j)}{k} - (\delta + r)p_K\right)k + rn - d(n)\right) \\
& + \zeta_g\left(n - v(n,s,g,j)\right) + \mathbf{1}_{g=1}\alpha_G\left(v(n,s,2,j) - v(n,s,1,j)\right) \\
& + \sum_{j'} \pi^J_{j,j'}\left(v(n,s,g,j') - v(n,s,g,j)\right) + \alpha_\star\left(v^\star + n - v(n,s,g,j)\right)
\end{aligned} \tag{4}
$$

- $d(n)$:exogenous dividend payout policy for the current level of net worth
- v_n: drift in net worth, which depends on the capital choice and dividend payout
- lifecycle:
  - ζ_g term captures firm exit
  - the transition from lifecycle state $g = 1$ to $g = 2$
- $\alpha_\star$ term captures the transition to an additional "superstar status",

Steady-state calibration details

- targets:
- s-type to target size distribution
- age-dependent exogenous exit rate to target age distribution (Andersen and Rozsypal, 2021)
- size x age distribution:
  - initial net worth of entrants (n_0) to target size of entrants
  - superstar productivity z^* : average employment of firms 20+

back

Cyclical calibration details

- targets: relative cyclicality by size and age, size of the recession, growth rate of young-small firms
- calibration instruments: two shocks + two features = 9 new parameters
- productivity penalty of entrants $y_1^G < 1$ to match average size of 0-aged firms
- only financial shocks:
  1. colateral ϕ_0 relative cyclicality of small entrants
  2. discount rate r_0 chosen such that recession delivers GDP fall of 5% on impact
- distribution of η - cyclicality of older firms
  1. $\eta_1 - \eta_3$: relative cyclicality of respective group
  2. η_4: average $\eta = 1$
- distribution of initial net worth
  1. $n_2 - n_4$: decling cyclicality of entrants
  2. n_1: average employment growth of smallest entrants

back

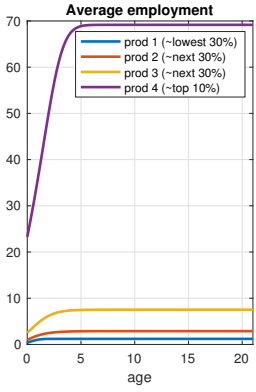
Model equations

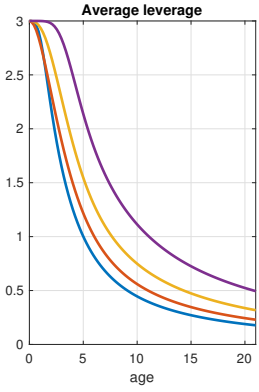
- final good manufacturer:
  - production function $Q = \left(\int_0^G q_i^\theta \, \mathrm{d}i \right)^{\frac{1}{\theta}}$
  - profits $\pi = \left(\int_0^G q_i^\theta \, \mathrm{d}i \right)^{\frac{1}{\theta}} - \int_0^G p_i q_i \, \mathrm{d}i$
  - FOC $q_i = p_i^{-\epsilon} Q$
  - zero profits $Q = \int_0^G p_i q_i \, \mathrm{d}i$
- aggregation accounting
  - final goods resource constraint: $Q = C + I + M$
  - aggregate output $Y = C + I + G$
  - intermediate goods $M = \int_0^G m_i di$.
- intermediate good production
  - production function: $q_i = z_i f(k_i, l_i, m_i)^{\eta_i}$
  - revenue: $p_i q_i = z^\theta f(k_i, l_i, m_i)^{\eta\theta} Q^{1-\theta}$
  - value added: $y = p_i q_i - m_i$
  - profit: $\pi(k, z) = \max_{l,m \geq 0} z^\theta f(k, l, m)^{\eta_z \theta} Q^{1-\theta} - wl - m$
  - capital evolution $\dot{k} = i - \delta k$

	Interpretation	Value	Source
r	Discount rate	0.0202	2% yearly real interest rate
z	Productivity distribution	-	See text
η_z	Returns to scale distribution	-	See text
ζ_z	Exit rates	-	See text
θ	Substitution across varieties	0.9	10% markup in frictionless model
α	Labor-capital ratio in prod fun	7.208	Aggregate L
β	Intermediate-capital ratio in prod fun	0.3703	Aggregate M
μ_0	Firm entry rate	0.0834	Normal total mass of firms to one
$\bar{\phi}$	S.s. collateral limit	3	Maximum leverage
δ	Depreciation rate	0.1054	10% annual rate
$\bar{n}$	Net worth where start paying dividends	38.78	Normalisation
α_s	Rate transition to superstar firm	5e-5	0.5% of firms are superstar
z_s	Superstar productivity	1.2803	Employment share of firms age 20+
χ	Labor disutility shifter	0.0128	Labor share of income
η	Labor supply elasticity	0.5	Real wage flexibility

Model results

Firm evolution by age





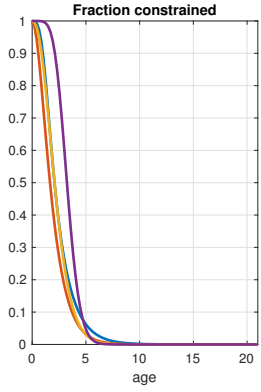
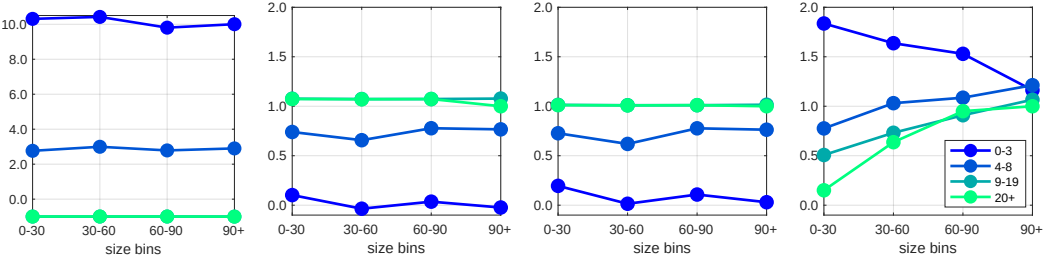


Figure: Cyclicality of employment

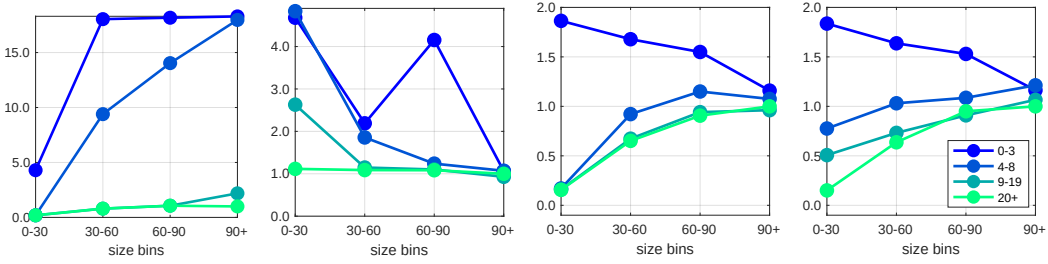


(a) Collateral ($\bar{\phi}$) (b) Discounts (r) (c) TFP (z) (d) Data

(normalised relative to oldest-largest (age 20+ size 90+) bin)

back

Figure: Cyclicality of employment



(a) Het. η_s (b) Het. n_s^e (c) Full model (d) Data

back

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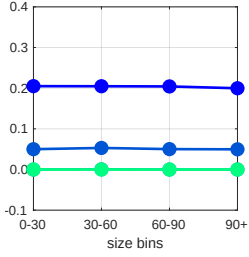
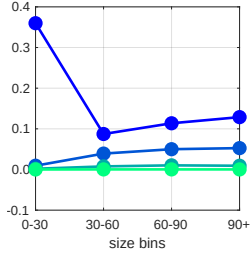
Model results

Growth rates

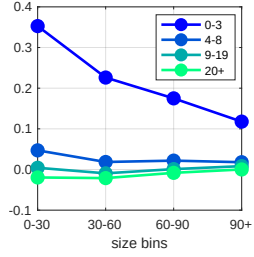
Figure: Average growth rate of employment



(a) Basic model



(b) Full model



(c) Data

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Model - firm shocks

- Heterogeneous productivity: $z \equiv z(s,q) = z_s^S z_g^G$
  - z^S: firm "quality" $\rightarrow$ contributes to size dispersion
  - z^G: age component $\rightarrow$ penalty for entrants
- Timing of shocks:
  - at entry:
    - z_s^S and η (currently perfectly correlated)
  - every period:
    - age specific exogenous exit
    - transition to superstar state
      - new (much larger) z_i and η_i
      - allowed to issue equity, become unconstrained

Calibration

1. Steady state
   - Size / age
   - Number of firms / Employment
2. Cyclicality
   - young vs old
   - small vs large

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	Size	Age
Number of firms	by construction (quantiles)	3 parameters for exit rate
Employment	z_i: average employment in each size bin	1: entrants' net worth 2: old firms' fraction of employment

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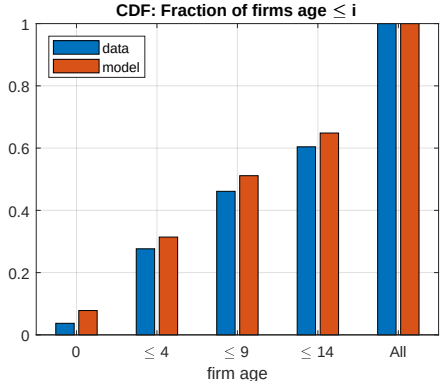
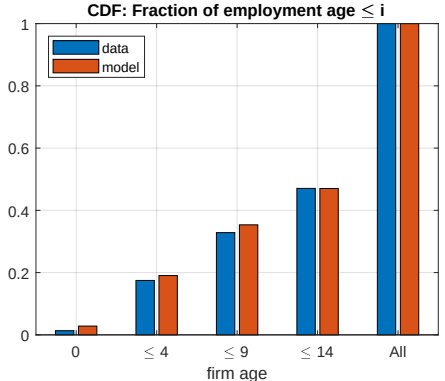


Model

Calibration targets: Steady state

	Size	Age
Number of firms	by construction (quantiles)	3 parameters for exit rate
Employment	z_i: average employment in each size bin	1: entrants' net worth 2: old firms' fraction of employment



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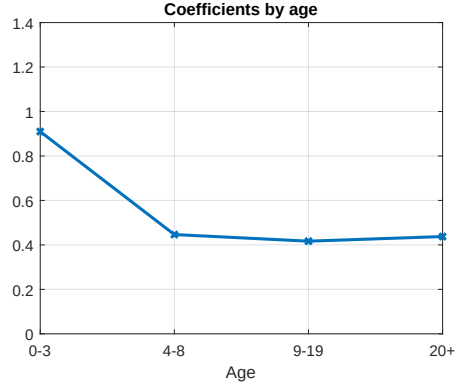
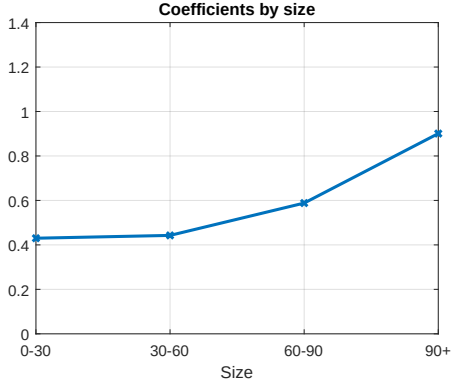


Calibration

1. Steady state
   - Size / age
   - Number of firms / Employment
2. Cyclicality
   - young vs old: size of the financial shock (λ)
   - small vs large: returns to scale (η_1)



Parametrisation

Targets: steady state

	Fraction of firms				Average employment			
Size	0-30	30-60	60-90	90+	0-30	30-60	60-90	90+
Model (s.s. cali)	0.30	0.30	0.30	0.10	2.01	5.85	16.61	138.02
Model (b.c. cali)	0.30	0.30	0.30	0.10	1.98	5.93	16.58	137.82
Data	0.36	0.26	0.28	0.10	1.95	5.65	15.90	146.15

(a) Size distribution

	Fraction of firms					Average employment				
Age	0	1-3	4-8	9-19	20+	0	1-3	4-8	9-19	20+
Model (s.s. cali)	0.08	0.19	0.21	0.27	0.25	9.41	13.63	18.54	20.25	33.41
Model (b.c. cali)	0.08	0.19	0.21	0.27	0.25	9.40	12.57	17.82	21.91	32.97
Data	0.05	0.18	0.25	0.25	0.27	9.35	11.90	16.44	21.82	32.95

(b) Age distribution

Parametrisation

Targers: cyclicality

Moment	Data	Model	Error	Associated parameter
Average employment growth age 0-3, size 0-30%	0.33	0.33	1.17%	n_1^e
Relative cyclicality age 0-3, size 0-30%	1.36	1.36	−0.35%	$\bar{\phi}_0$
Relative cyclicality age 0-3, size 30-60%	1.37	1.31	−4.96%	n_2^e
Relative cyclicality age 0-3, size 60-90%	1.32	1.33	0.95%	n_3^e
Relative cyclicality age 0-3, size 90%+	0.95	1.05	9.47%	n_4^e
Relative cyclicality age 20+, size 0-30%	0.24	0.26	5.17%	η_1
Relative cyclicality age 20+, size 30-60%	0.64	0.65	1.87%	η_2
Relative cyclicality age 20+, size 60-90%	0.94	0.88	−5.92%	η_3
5% peak GDP fall during recession	-0.05	-0.05	−0.37%	r_0
Average error (sqrt. of mean squared error)	–	–	4.50%	–

Parametrisation

Common parameters

	Parameters used in both calibrations:			
r	Discount rate	0.0202	0.0202	2% yearly real interest rate
δ	Depreciation rate	0.1054	0.1054	10% annual rate
θ	Substitution across varieties	0.9	0.9	10% markup in frictionless model
α	Labor-capital ratio in prod fun	9.1331	8.4815	Aggregate L
μ_0	Firm entry rate	0.0834	0.0834	Normal total mass of firms to one
$\bar{\phi}$	S.s. collateral limit	3	3	Maximum leverage
$\bar{n}$	Net worth where start paying dividends	59.9283	84.3044	Normalisation
χ	Labor disutility shifter	0.0114	0.0114	Labor share of income
σ	Labor supply elasticity	0.3	0.3	Real wage flexibility
α_s	Rate transition to superstar firm	5.1e-05	5.1e-05	0.5% of firms are superstar
$z_\star$	Superstar productivity	0.6393	0.4768	Employment share of firms age 20+
ζ_y	Exit rate when young ($g = 1$)	0.1415	0.1415	Exit rate age 0
ζ_o	Exit rate when old ($g = 2$)	0.0647	0.0647	Average exit rate 8% per year
α_G	Transition rate young to old	0.1964	0.1964	Exit rate age 6
σ^I	Std. idiosyncratic shocks	0.0234	0.0234	Std. investment rates
ρ^I	Autocorr. idiosyncratic shocks	0.6590	0.6590	Khan and Thomas (2013)
z_1^S	Productivity for type $s = 1$	0.3288	0.3137	Av. emp. size 0-30%
z_2^S	Productivity for type $s = 2$	0.3681	0.3454	Av. emp. size 30-60%
z_3^S	Productivity for type $s = 3$	0.4103	0.4000	Av. emp. size 60-90%
z_4^S	Productivity for type $s = 4$	0.5035	0.4183	Normalise $Y = 1$
γ^S	Fraction born type $s = 1$	0.3	0.3	Firms for 0-30% size bin

Parametrisation

Calibration specific parameters

	Parameters used "Steady state" calibration:			
η	Returns to scale (all firms)	1	–	All firms CRS
n^e	Net worth fraction of entrants	0.3543	–	Average employment of age 0 firms
z_1^G	Relative productivity of young	1	–	Not used

	Parameters used in "Cyclical" calibration:			
η_1	Returns to scale ($s = 1$)	–	0.7952	SMM
η_2	Returns to scale ($s = 2$)	–	1.0407	SMM
η_3	Returns to scale ($s = 3$)	–	0.9887	SMM
η_4	Returns to scale ($s = 4$)	–	1.0407	Impose agg. economy has CRS
n_1^e	Net worth fraction of entrants ($s = 1$)	–	0.1937	SMM
n_2^e	Net worth fraction of entrants ($s = 2$)	–	0.4664	SMM
n_3^e	Net worth fraction of entrants ($s = 3$)	–	0.9188	SMM
n_4^e	Net worth fraction of entrants ($s = 4$)	–	0.8297	SMM
z_1^G	Relative productivity of young	–	0.9289	Average employment of age 0 firms
$\bar{\phi}_0$	Size of collateral constraint shock	–	-0.0926%	SMM
r_0	Size of discount rate shock	–	0.1562%	SMM

Empirical results overview

1. Cyclicality of employment, sales, debt and assets

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   - *on average*, only very young firms grow (strongly decreasing in size), firms above 10 shrink (weakly increasing in size)
   - Over age, both debt and assets increase

▸ results